

Golf Ball Projectile Motion Project

by

Kevin LaBeau
Thao Lai
Kelly Martin
Brian Stilley

School of Engineering
Grand Valley State University

EGR 312 – Dynamics
Instructor: Dr. Mohammadzadeh

November 17, 2004

Introduction

The projectile motion of a golf ball is affected by the presence of air resistance. Without air friction opposing the force on the ball from the golf club, the golf ball trajectory would be an ideal projectile motion curve. By studying the effects of air resistance on moving objects, actual motion curves can be predicted. MATLAB is a program that can implement numerical schemes, such as the method of central differences and fourth order Runge-Kutta. It aids in the study of drag force by creating visual simulations of the trajectories from differential equations.

Background

Drag force in projectile motion is the result of interactions between air molecules opposing the motion of solid objects. This physical phenomenon is commonly referred to as air friction. Drag force acting on a golf ball in projectile motion can be calculated using Equation (1), where D is the drag force acting on the ball, V is the speed of the ball, ρ is the density of air, A is area of the projectile interacting with air, and C_D is the drag coefficient.

$$D = \frac{1}{2} C_D \rho A V^2 \quad (1)$$

Assuming the golf ball travels at high speed, the drag coefficient is a function of air viscosity μ , air density ρ , golf ball diameter d , and ball speed V . Consequently, the drag coefficient can be represented by Equation (2). Higher viscosities, or frictional effects of fluid on the solid body, yield higher frictional force on the adjacent body. The effects of density, air viscosity, ball speed, and ball diameter on the drag coefficient is combined into a dimensionless parameter known as Reynolds Number, Re in (3).

$$C_D = f(\mu, \rho, d, V) \quad (2)$$

$$Re = \frac{\rho V d}{\mu} \quad (3)$$

Therefore, the drag coefficient is defined as a function of Reynolds Number.

$$C_D = f(Re) \quad (4)$$

Drag force inherently opposes the direction of the velocity of the golf ball. With the assumption that the path of travel is in the two-dimensional x - y coordinate system, the motion is modeled by differential Equations (5) through (8). In these equations, t is the time of flight, u and v are components of projectile velocity in the x and y directions, respectively, g is the gravitational acceleration, and τ is the time constant.

$$\frac{dx}{dt} = u \quad (5)$$

$$\frac{dy}{dt} = v \quad (6)$$

$$\frac{du}{dt} = -\frac{C_D \text{Re}}{24\tau} u \quad (7)$$

$$\frac{dv}{dt} = -\frac{C_D \text{Re}}{24\tau} v - g \quad (8)$$

The time constant, τ , can be defined as Equation (9) with m representing the mass of the golf ball.

$$\tau = \frac{m}{3\pi d\mu} \quad (9)$$

Problem Statement

The projectile motion of the golf ball depends on the constants that are given in Table 1.

Table 1: Given Constants

Parameter	Variable	Value
Weight (oz)	w	1.5
Ball diameter (inches)	d	1.75
Air viscosity (lb-sec/ft)	μ	0.376×10^{-6}
Density (slugs/ft ³)	ρ	0.002378

The ball leaves the ground with an initial velocity, V_o , of 120 ft/sec² at an angle of 30 degrees from the horizontal direction. It is assumed that the drag coefficient, C_D , is 0.4 when the

Reynolds Number is less than or equal to 90,000 and C_D is 0.1 when the Reynolds Number is greater than 90,000.

- 1) Derive the differential equations of motion, (5) through (8).
- 2) Assume no friction in the projectile motion, solve for x , y , u , and v for the duration of flight ($y \geq 0$), symbolically, and then plot the path of the golf ball.
- 3) Assume that the product $C_D Re$ in equations (7) and (8) is constant, evaluated at $Re = 9 \times 10^4$ with $C_D = 0.25$, then solve the system of differential equations. Plot the path of the ball.

If the air friction is not constant, the system of differential equations should be solved numerically. Use two numerical schemes to obtain the path of the ball.

- 4) Use the method of central differences to obtain the path of the ball. In this method, approximate the variables and their derivatives in equations (5) through (8). Rewrite the equations as follows:

$$x(t) = x(t - \Delta t) + \frac{\Delta t}{2} [u(t - \Delta t) + u(t)] \quad (10)$$

$$y(t) = y(t - \Delta t) + \frac{\Delta t}{2} [v(t - \Delta t) + v(t)] \quad (11)$$

$$u(t) = u(t - \Delta t) - \frac{C_D Re}{24\tau} \left[\frac{u(t) + u(t - \Delta t)}{2} \right] \Delta t \quad (12)$$

$$v(t) = v(t - \Delta t) - \frac{C_D Re}{24\tau} \left[\frac{v(t) + v(t - \Delta t)}{2} \right] \Delta t - g\Delta t \quad (13)$$

Solve for $u(t)$ and $v(t)$ from equations (10) through (11).

- 5) Use the fourth-order Runge-Kutta method to obtain the path of the ball. The function, `ode45` in MATLAB should be used to solve for u , v , x , and y .

Solution

Before solving for values in any of the problems, a free body diagram and effective force diagram were drawn to relate the forces acting on the golf ball. These are shown in Figure 1.

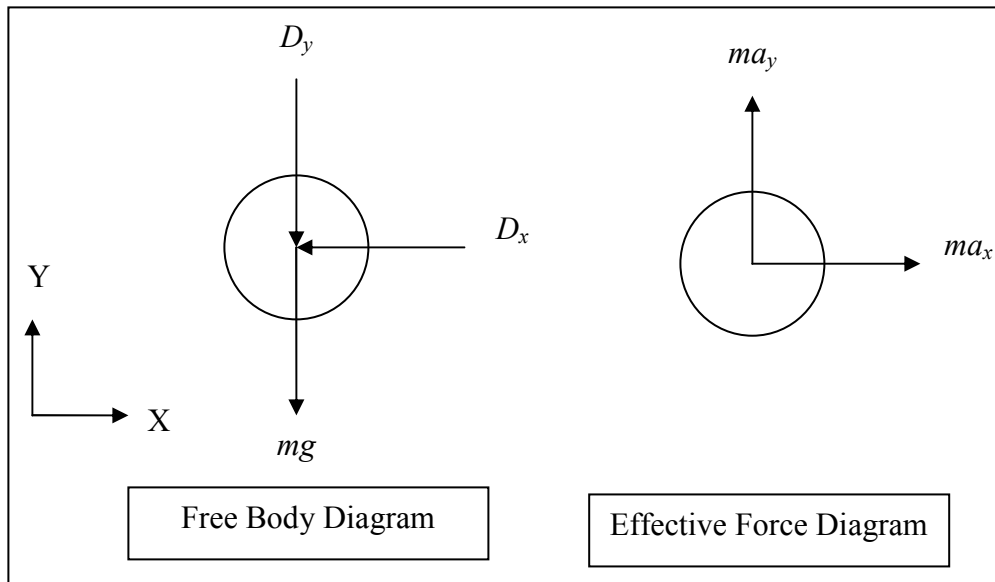


Figure 1: Free body and effective force diagrams of a golf ball projectile

- 1) With the defined constants and their relationships, the equations of motion were derived in Appendix A – Part 1. The derivations resulted in Equations (5) through (8).

$$\frac{dx}{dt} = u \quad (5)$$

$$\frac{dy}{dt} = v \quad (6)$$

$$\frac{du}{dt} = -\frac{C_D \text{Re}}{24\tau} u \quad (7)$$

$$\frac{dv}{dt} = -\frac{C_D \text{Re}}{24\tau} v - g \quad (8)$$

- 2) Assuming that the golf ball experiences no friction in the projectile motion, x , y , u , and v were symbolically solved for the duration of the flight. The steps of these

calculations can be found in Appendix A - Part 2. The position and velocity functions that were developed resulted in Equations (14) through (17).

$$x = x_0 + u(t) \quad (14)$$

$$y = y_0 + v(t) - \frac{1}{2} g(t)^2 \quad (15)$$

$$u = u_0 = \text{const.} \quad (16)$$

$$v = v_0 - g(t) \quad (17)$$

The kinematic equations were implemented in a MATLAB program, found in Appendix B, which plotted the projectile motion.

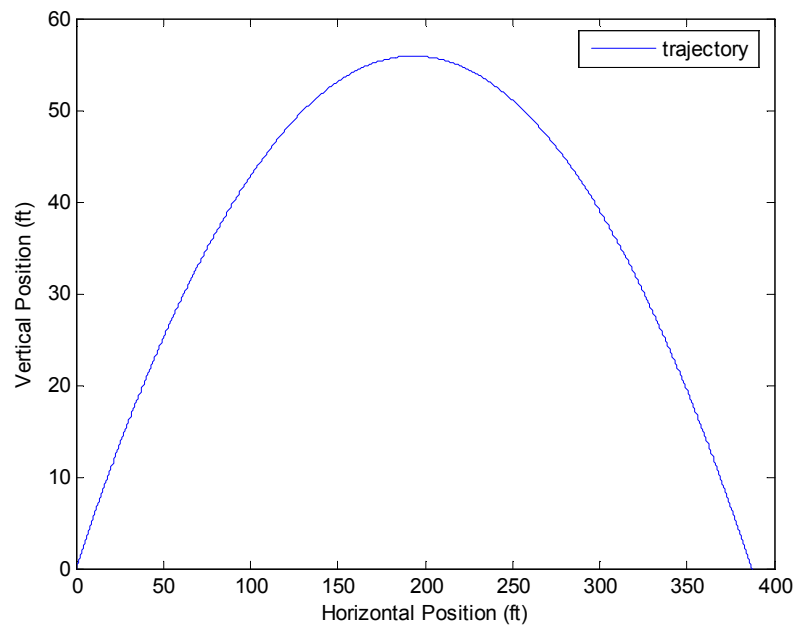


Figure 2: Golf ball projectile motion without the effects of drag force

Table 2: Parameters of Golf Ball at End of Trajectory - No Drag Force

Variable	Final value
x	387.32 ft
y	55.90 ft
u	103.92 ft
v	-60.01 ft
t	3.73 s

The path of golf ball without the effects of air drag resulted in a parabolic trajectory. The values in Table 2 indicate that the x velocity never decreased. The ball fell due to the effect of gravity in the y direction. Therefore, the range and maximum height of the ball in this situation are not possible in the real world. The values given in Table 2 may not match the expected results exactly because they are approximations based on calculations using the central differences method.

- 3) The product $C_D Re$ in Equations (7) and (8) was assumed constant, where $Re = 9 \times 10^4$ and $C_D = 0.25$. The system of equations, Equations (5) through (8), were plotted to find the path of the ball. The constants found in Table 1 were substituted into the equations when the initial velocity, V_0 , was 120 ft/sec at an angle of 30° . Plugging those values into the state equations resulted in Equations (18) through (21).

$$u = u_0 e^{-\frac{C_D Re}{24\tau} t} \quad (18)$$

$$v = \left(v_0 + \frac{g 24\tau}{C_D Re} \right) e^{-\frac{C_D Re}{24\tau} t} - \frac{g 24\tau}{C_D Re} \quad (19)$$

$$x = -\frac{24\tau}{C_D Re} u_0 e^{-\frac{C_D Re}{24\tau} t} + \frac{24\tau}{C_D Re} u_0 \quad (20)$$

$$y = \left(-\frac{24\tau v_0}{C_D Re} - \left(\frac{24\tau}{C_D Re} \right)^2 g \right) e^{-\frac{C_D Re}{24\tau} t} - \frac{g 24\tau}{C_D Re} t + \frac{24\tau v_0}{C_D Re} + \left(\frac{24\tau}{C_D Re} \right)^2 g \quad (21)$$

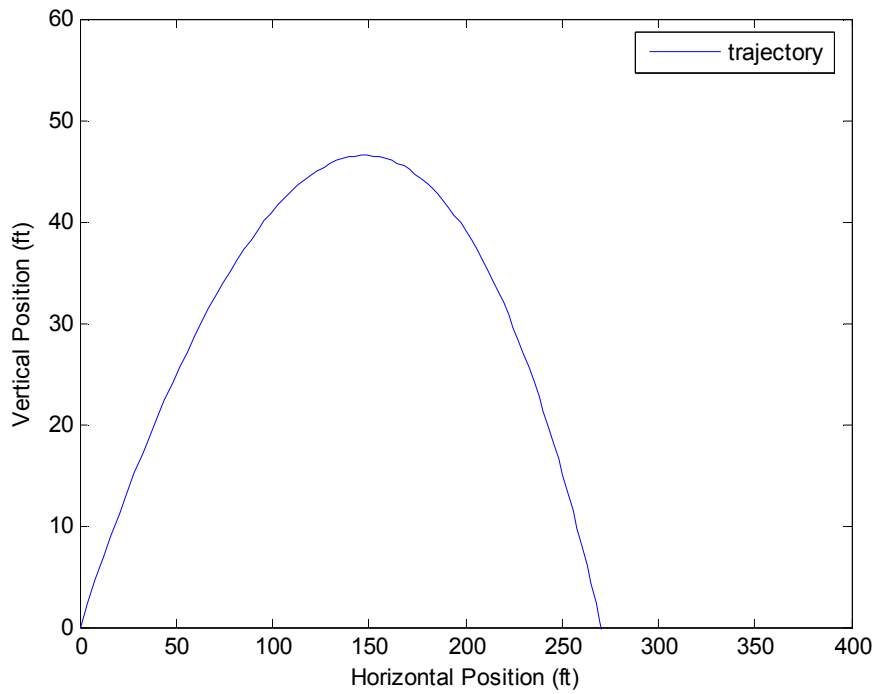


Figure 3: Golf ball projectile motion with constant drag force

Table 3: Parameters of Golf Ball at End of Trajectory - Constant Drag Force

Variable	Final value
x	272.38ft
y	46.52 ft
u	N/A
v	N/A
t	3.44s

The u and v values for part 3 were not calculated in the program.

The drag force opposing the direction of motion, resulted in a shorter range and maximum height than the trajectory with no friction. Consequently, the time in flight of the ball was less when drag force is applied. The shape of the trajectory was parabolic because the drag force was constant. The values in Table 3 are approximations calculated using the method of central differences.

- 4) The derivatives of equations (5) through (8) were approximated in equations (10) through (13). To solve these equations using the method of central differences, equations (10) and (11) were used in their given form, but equations (12) and (13) were manipulated algebraically in order to isolate $u(t)$ and $v(t)$ on one side of the equation and get them in their simplified form shown in equation (22) and (23). The MATLAB program implementing the method of central differences can be found in Appendix A - Part 4. The resulting plot of the golf ball can be found in Figure 4.

$$u(t) = \frac{48\tau(u(t - \Delta t)) - C_D \operatorname{Re}(u(t - \Delta t))\Delta t}{48\tau + C_D \operatorname{Re} \Delta t} \quad (22)$$

$$v(t) = \frac{48\tau(v(t - \Delta t)) - C_D \operatorname{Re} dv(t - \Delta t) - 48\tau g dt}{48\tau + C_D \operatorname{Re} dt} \quad (23)$$

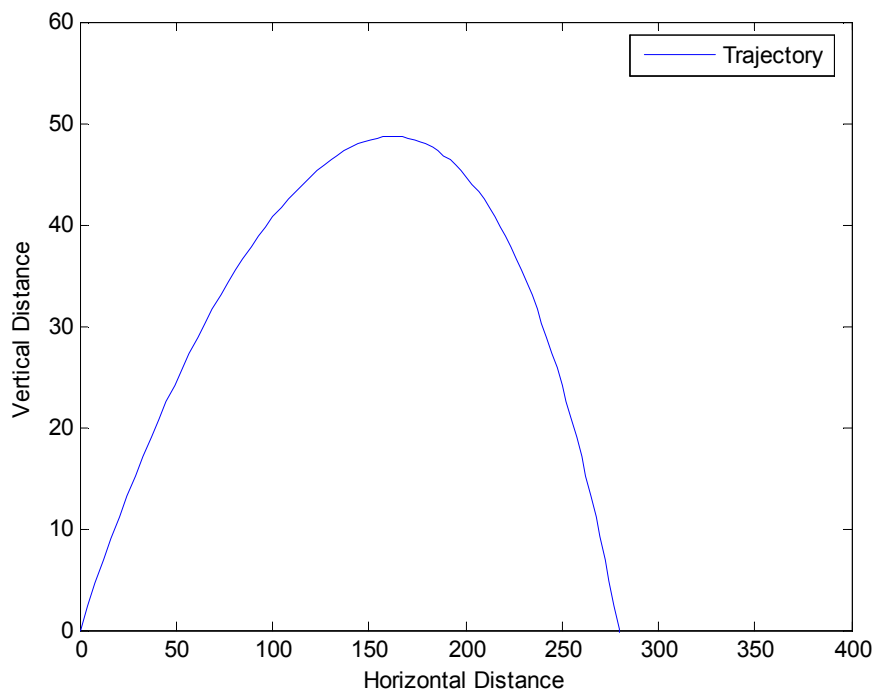


Figure 4: Golf ball projectile motion with varying drag force using method of central differences

Table 4: Parameters of Golf Ball at End of Trajectory - Varying Drag Force

Variable	Final value
x	281.72 ft
y	48.68 ft
u	58.57 ft
v	-61.48 ft
t	3.44 s

The velocity in the x and y directions decreased at varying rates, resulting in a leaning parabolic path. The range and maximum values were less than those of part 2, but greater than those of part 3 because the model allowed the friction force to vary with the speed of the ball, while the constant force was simply an average.

- 5) The path of the golf ball was plotted using a fourth-order Runge-Kutta numerical analysis. Equations (5) through (8) were implemented in the MATLAB programs found in Appendix B, as Figure B4 and Figure B5. The resulting golf ball projectile motion, when the air resistance was not constant, is shown in Figure 5.

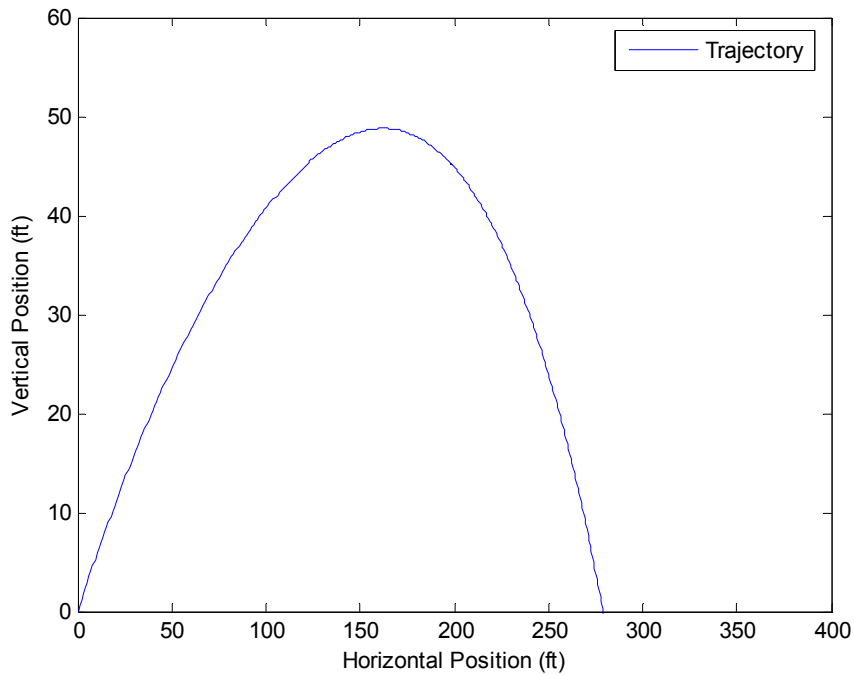


Figure 5: Golf ball projectile motion with varying drag force using fourth-order Runge-Kutta

Table 5: Parameters of Golf Ball at End of Trajectory - Varying Drag Force

Variable	Final value
x	281.72 ft
y	48.68 ft
u	58.57 ft
v	-61.48 ft
t	3.44 s

The model for part 5 is the same as part 4, therefore the trajectory is also the same. Runge Kutta is a more precise numerical method than Central Differences Method, but both yielded the same results. The time step in the Central Differences program was very small, which can explain the matching answers.

While comparing the various projectile motion plots from each analysis, a number of conclusions can be formed. When air resistance is neglected and gravity is the only acting force on the golf ball, it can travel in an ideal projectile path. This condition is not the case in the real world. From the other plots, that do take drag force into account, the motion of the golf ball resembles a more realistic plot where the velocity decreases at varying rates. The golf ball does not travel as far due to this air friction.

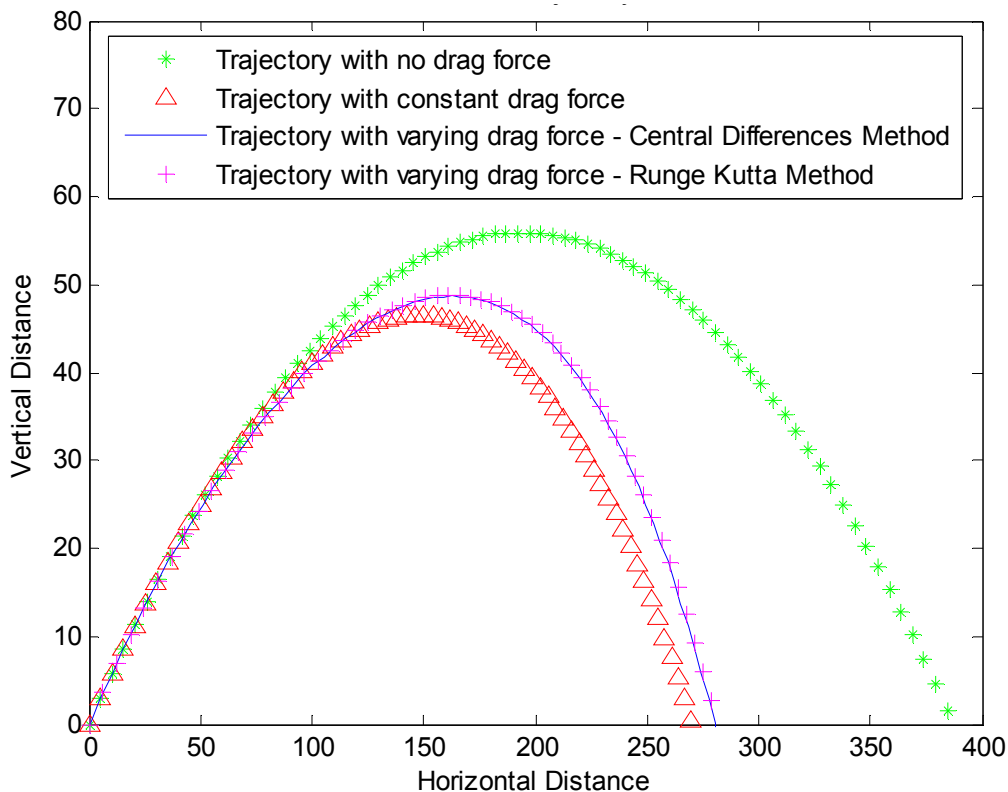


Figure 6: Golf ball projectile motion comparison

Conclusions

Golf ball trajectory is contingent on the amount of force applied to it and the amount of air resistance that it experiences while in motion. The projectile motion of the ball would be perfect without air resistance; specifically, the maximum height of the travel would occur precisely between the initial and final position. In real life situations, air resistance must be considered in the path of the golf ball in order to accurately predict how and where the ball will

travel. By modeling system equations for the trajectory in a MATLAB program, the projectile motion simulations can be developed for various environmental conditions.